5a) Reduction formula (7M) (6M)

① $\int \sin^n x\,dx$

Let $I_n = \int \sin^n x\,dx$.

where n is positive Integer.

$$I_n = \int \sin^{n-1} x\,(\sin x)\,dx$$

Applying the method of integration by parts, we get

$$I_n = \sin^{n-1} x\,(-\cos x) - \int (n-1)\sin^{n-2} x(\cos x)(-\cos x)\,dx.$$

$$= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\cos^2 x\,dx.$$

$$= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\left[1-\sin^2 x\right]dx.$$

$$= -\sin^{n-1} x\cos x + (n-1)\int \left(\sin^{n-2} x - \sin^n x\right)dx$$

$$= -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$$

$$I_n = -\sin^{n-1} x\cos x + (n-1)I_{n-2} - (n-1)I_n$$

$$I_n + (n-1)I_n = -\sin^{n-1} x\cos x + (n-1)I_{n-2}$$

$$I_n\left[1+(n-1)\right] = -\sin^{n-1} x\cos x + (n-1)I_{n-2}$$

$$nI_n = -\sin^{n-1} x\cos x + (n-1)I_{n-2}$$

$$I_n = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}I_{n-2}$$

This is the reduction formula for $\int \sin^n x\,dx$.